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Proposed Approximation Algorithm for Vehicle Routing Problem using Self-Organizing Maps

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Abstract: The distribution industry must raise distribution efficiency in an attempt to solve numerous issues including labor shortages, surging fuel costs, and environmental impacts. The vehicle routing problem (VRP) is a combinatorial optimization problem for simultaneous determination of vehicle loading limits, assignment of delivery locations to each vehicle, and travel routes of each vehicle. This simultaneous determination makes it difficult to ascertain the optimal solution to the problem on a practical scale. Therefore, an approximate method is used. A self-organizing map (SOM) is a type of approximate method for finding an effective solution to the travelling salesman problem (TSP), which is a fundamental problem of vehicle routing. As a solution to the VRP, this study proposes a method (SOM+OP+2-opt solution) to find an efficient travel route for each vehicle by finding travel routes to all delivery locations using an SOM at the first stage and applying the optimal partitioning (OP) and 2-opt methods to travel routes found at the second stage. As a numerical experiment, the study compares a basic method (sweep +2-opt solution) and proposed methods to verify the effectiveness of the proposed methods.

Keywords: 2-opt Method, Optimal Partitioning Method, Self-Organizing Map, Sweep Method, Traveling Salesman Problem, Vehicle Routing Problem

1. Introduction

The distribution industry today faces/confronts labor shortages and vehicle drivers' aging, in addition to persistent work environment issues such as long working hours and low wages. Moreover, increased redeliveries are occurring because of the growth of the e-commerce markets. Surges in expenses for fuel and vehicle maintenance are raising distribution costs. Therefore, a distribution system that reduces environmental loads and which also increases efficiency must be developed urgently. The VRP (Bowerman, Paul, Calamai & Hall, 1994; Toth & Vigo, 2015) is a combinatorial optimization problem for simultaneous determination of vehicle loading limits, assignment of delivery locations to each vehicle, and travel routes of each vehicle for multiple delivery locations. Because determination of the optimal solution to the problem on a practical scale is extremely difficult, an approximate method is used to find an approximate solution. A self-organizing map SOM (Kohonen, 2001) is a type of approximation method that identifies, through numerical experimentation, an effective solution to the travelling salesman problem (TSP) (Lin & Kernighan, 1973), which is a basic problem of vehicle routing (Angeniol, Vaubois & Texier, 1988; Takano, Shirai & Matsumoto, 2011).

As a solution to VRP, this study proposes a method (SOM+OP+2-opt solution) of finding travel routes to all delivery locations using an SOM at the first stage, at the second stage, among the travel routes found, the method determines an efficient travel route for each vehicle using optimal partitioning (OP) (Bowerman, Paul, Calamai & Hall, 1994) and 2-opt (Englert, Röglin & Vöcking, 2016). As a numerical experiment, the study compares a basic method (sweep +2-opt solution) and the proposed methods to verify the effectiveness of the proposed methods.

2. Vehicle Routing Problem (VRP)

The VRP is a problem that demands identification of the route with the shortest total delivery distance among a set of routes to visit all delivery locations once using multiple vehicles under given constraints. Such constraints include a “loading

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limit," prohibiting each vehicle from delivering goods weighing more than the loading limit at a time, "assignment restriction," limiting vehicles going to each delivery location to only one, and a "travel route restriction," specifying each vehicle to depart from a distribution center, go around delivery locations, and return to the distribution center. The following presents the VRP formulation.

Minimization

$$Z = \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} x_{kij} d_{ij} \qquad (i \neq j) \tag{1}$$

Subject to (constraints)

$$\sum_{i=0}^{n} \sum_{j=1}^{n} w_j x_{kij} \leq C_k \quad (\forall k \in K\ ; i \neq j) \tag{2}$$

$$\sum_{k=1}^{m} x_{kij} = 1 \quad (\ \forall i, j \in N, ; i \neq j) \tag{3}$$

$$x_{kij} \in \{0,1\} \quad \forall i, j \in N, \forall k \in K \tag{4}$$

Z: total delivery distance n: number of delivery locations m: number of vehicles
d_{ij}: distance from delivery location i to deliver location j
x_{kij}: =1 when vehicle k travels the route from distribution i to deliver location j
=0 for other cases
w_i: demand at distribution i C_k: loading limit at vehicle k
N: set of delivery locations = $\{0, 1, 2, \cdots, n\}$ (0: distribution center)
K: set of delivery vehicles =$\{1, 2, \cdots, m\}$

Equation (1) represents minimization of the total delivery distance Z. Equation (2) expresses a loading limit that prohibits a vehicle from delivering goods weighing more than its loading limit at a time. Equation (3) shows an assignment restriction by which only one vehicle delivers goods to each delivery location. Equation (4) shows that x_{kij} is a 0–1 variable.

3. Methodology

3.1. Overview of self-organizing maps (SOM)

An SOM is a type of neural network and a two-layer unsupervised competitive learning model without a middle layer (Kohonen, 2001). The SOM learning algorithm learns characteristics of the input data through unsupervised competitive neighborhood learning. Based on this, the SOM creates a map by placing data with similar characteristics in mutual proximity and other distant data. This mapping allows the SOM to visualize high-level information and facilitates people's intuitive understanding of relations among data. Angeniol et al. (Angeniol, Vaubois & Texier, 1988) were the first to apply the TSP to an SOM. This algorithm presented by Angeniol et al. uses a method that starts with one initial node. While adding and reducing nodes as necessary, it brings them closer to a delivery location, eventually making them one to one. Figure 1 exhibits a conceptual diagram of node movement on an SOM.

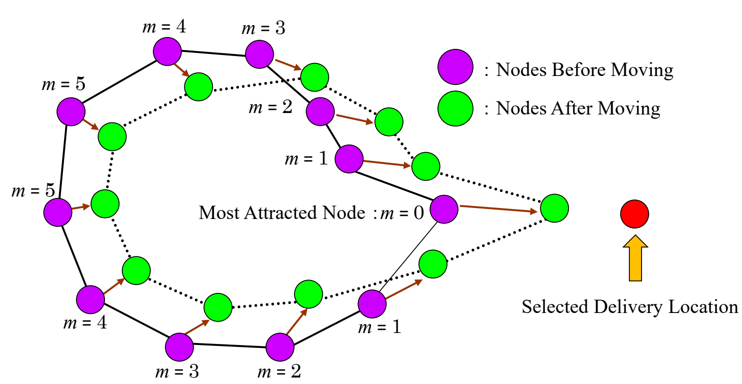


Figure 1. Conceptual diagram of node movement on SOM.

3.2. Overview of sweep method, 2-opt method, and partitioning method

The sweep method (Renaud & Boctor, 2002) is a heuristic technique used for the VRP. Starting from a distribution center, this method converts the position of each delivery location to polar coordinates (angle and distance), arranges delivery locations from the smallest angle, groups them in order not to exceed vehicle loading limits, and determines the travel route of each group.

The 2-opt method is a local search technique to improve routes for the TSP and VRP. This method selects two from available routes, exchanges the two routes, and compares the travel distance before and after the exchange. If the travel distance becomes shorter, then the travel route after the exchange is selected as a new one.

The partitioning method is a technique for solving large optimization problems such as the TSP and VRP by dividing them into small partial problems. The method groups delivery locations geographically or based on demand to determine the travel route of each group.

3.3. Basic method (sweep + 2-opt solution)

The basic method, at the first stage, is to partition (assign) delivery locations that fan out from a distribution center by following the angles of the distribution center and delivery locations, considering vehicle's loading limits. At the second stage, it raises the efficiency of travel routes to deliver locations assigned to each vehicle using the 2-opt method. The following explains the algorithm of the basic method. Figure 2 is a representation of the basic method.

[Step 1] Arrange all delivery locations from the distribution center in order from the one with the smallest angle.
[Step 2] Select a delivery location that will be the starting point of partitioning.
[Step 3] By following the angles from the starting point and distribution center, assign a vehicle to a delivery area that extends to the delivery location immediately before the delivery location in which the total amount of demand exceeds the vehicle loading limit. Repeat this process until vehicles are assigned to all delivery locations.
[Step 4] Determine travel routes from the distribution center to deliver locations within the delivery areas of the assigned vehicles and increased efficiency of the travel routes using the 2-opt method.
[Step 5] Change the delivery location that becomes the starting point of partitioning and repeat [Step 3] through [Step 4].
[Step 6] When all delivery locations are selected as a starting point of partitioning, rearrange the delivery locations arranged in Step 1 in reverse order and repeat the process from Step 2 through Step 5.
[Step 7] Finish the process when all delivery locations placed in reverse order have been selected as a starting point with the shortest travel route obtained from the search as the solution.

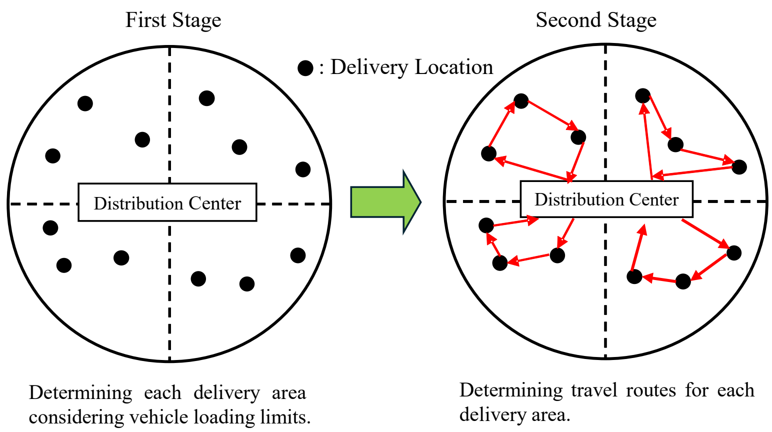


Figure 2. Image showing the basic method

3.4. Proposed method (SOM+OP+2-opt solution)

The proposed method is, at the first stage, to determine travel routes to all delivery locations other than the distribution center using the SOM, and at the second stage, to create each vehicle's travel route to the extent that the total amount of demand at delivery locations does not exceed the vehicle's loading limit while maintaining the travel routes obtained at the first stage, thereby raising the efficiency of the travel routes using the 2-opt method. The following explains the algorithm of the proposed solution. Figure 3 depicts an image of the proposed method. This study also examines cases of using and not using the 2-opt method in the proposed methods.

[Step 1] Find a travel route that visits all delivery locations other than the distribution center once using the SOM.
[Step 2] Determine the starting points of the travel routes obtained in Step 1 and assign each vehicle to a delivery area covering from its starting point to the delivery location immediately before the delivery location in which the total amount of demand exceeds the vehicle loading limit. Connect the start and end of a route with the distribution center to form a travel route. The next starting point is the delivery location immediately after the ending point.
[Step 3] Repeat Step 2 until vehicles are assigned to all delivery locations.
[Step 4] Increase the efficiency of travel routes to which vehicles are assigned using the 2-opt method.
[Step 5] Change the starting point of the travel route obtained in Step 1 to the next delivery location and perform Step 2 through Step 4.
[Step 6] When all delivery locations are selected as a starting point, reverse the travel route obtained in Step 1 and perform Step 2 through Step 5.
[Step 7] Finish the process when all delivery locations on the reverse travel route have been selected as a starting point, with the shortest travel route obtained from the search as the solution.

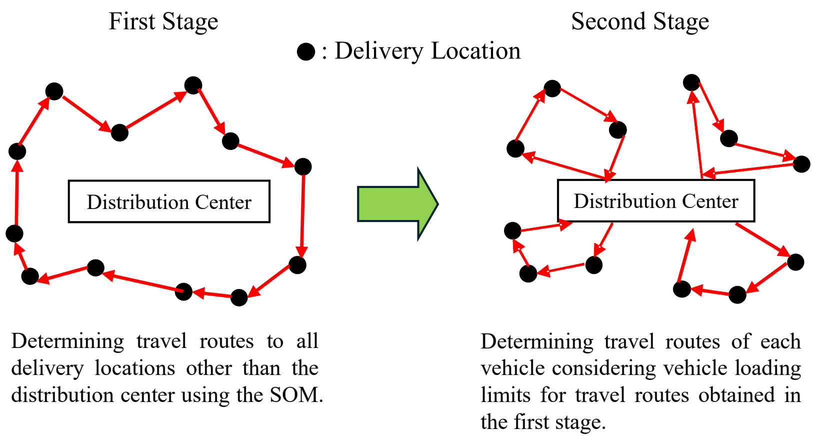


Figure 3. Image showing the proposed methods.

4. Numerical Experiment

4.1. Experiment conditions

Performance of the basic method and proposed methods (with and without the 2-opt method) is assessed under delivery conditions that the vehicle loading limits are 100, 150, and 200 to the benchmark problem (TSPLIB) of delivery locations 50, 75. and 100 of VRP. The number of trials for the proposed methods (with and without the 2-opt method) is set to 10.

4.2. Experiment result

Table 1 presents the total delivery distances when using the basic method and the cases of 10 trials of the proposed methods (with and without the 2-opt method). Table 2 shows the standard deviations of delivery distances of each vehicle. The colored cells indicate the minimum value under each set of delivery conditions. Under the conditions of delivery location 50 and load capacity 200, Figure 4 shows the travel routes obtained using the basic method, Figure 5 depicts the travel routes obtained using the proposed method with the 2-opt method. The value "0" in the figures denote the distribution center (at the center of the figures). The other values represent delivery location numbers.

Table 1. Total delivery distance

Delivery Locations	Vehicle Loading Limits	Total Delivery Distance						
		Basic Method	Proposed Method (without the 2-opt Method)			Proposed Method (with the 2-opt Method)		
			Maximum	Average	Minimum	Maximum	Average	Minimum
50	100	696.07	753.58	738.14	714.01	713.74	701.20	691.58
	150	569.68	635.96	624.05	611.68	605.10	589.75	579.01
	200	526.23	590.93	570.31	538.65	560.33	538.17	514.87
75	100	1141.94	1174.53	1146.33	1129.70	1114.35	1098.03	1077.30
	150	874.61	933.75	919.03	897.40	883.37	868.50	849.98
	200	784.00	839.33	828.21	808.29	789.17	774.81	762.36
100	100	1264.85	1370.00	1344.87	1330.23	1278.58	1260.99	1237.11
	150	1009.49	1120.72	1097.99	1069.87	1027.81	1010.85	992.11
	200	906.70	988.30	966.31	950.97	902.93	892.76	872.91

Table 2. Standard deviation of delivery distances per vehicle

Delivery Locations	Vehicle Loading Limits	Standard Deviation of Delivery Distances per Vehicle						
		Basic Method	Proposed Method (without the 2-opt Method)			Proposed Method (with the 2-opt Method)		
			Maximum	Average	Minimum	Maximum	Average	Minimum
50	100	13.042	28.969	24.959	21.075	29.726	25.552	21.527
	150	21.249	38.195	31.112	22.206	34.381	26.432	18.263
	200	10.393	51.364	25.717	7.494	47.344	23.182	9.659
75	100	10.994	29.178	24.467	20.137	26.212	23.788	20.250
	150	12.059	31.772	26.295	23.951	29.533	26.464	24.772
	200	26.865	41.776	34.295	20.013	39.691	35.149	28.957
100	100	14.522	27.503	23.696	21.200	26.603	22.892	20.366
	150	30.274	35.401	30.208	22.380	35.415	28.591	18.787
	200	25.818	32.547	28.095	19.953	30.550	27.172	24.000

A comparison of total delivery distances between the proposed method without the 2-opt method and the proposed method with the 2-opt method in Table 1 revealed the proposed method with the 2-opt method as a better solution under all delivery conditions. This result suggests that the efficiency of the travel routes raised by the 2-opt method was effective. A comparison of total delivery distances between the basic method and proposed method with the 2-opt method revealed that the total delivery distance was reduced through the basic method under one condition and through the proposed method with the 2-opt method under eight of nine delivery conditions. The proposed method with the 2-opt method led to results which were better than those obtained using the basic method under many delivery conditions through vehicle assignment not possible with the basic method. Moreover, the number of vehicles was reduced under some delivery conditions.

Comparisons of standard deviation of delivery distances per vehicle among the basic method, the proposed method with the 2-opt method, and the proposed method without the 2-opt method revealed that the standard deviation out of nine delivery conditions was reduced under four conditions in the case of the basic method, under three conditions in the case of the proposed method without the 2-opt method, and under two conditions in the case of the proposed method with the 2-opt method. Dispersion of vehicles' delivery distances in the proposed methods generally tends to be larger than that of the basic method.

Numerical experimentation showed that the proposed methods reduced the total delivery distance more than the basic method, verifying the effectiveness of the proposed methods. Vehicles' delivery distances must be made as equal as possible because of the dispersion which exists among them.

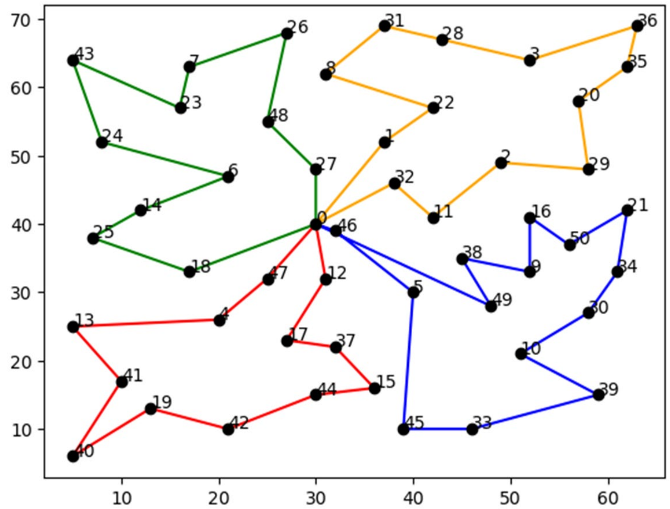
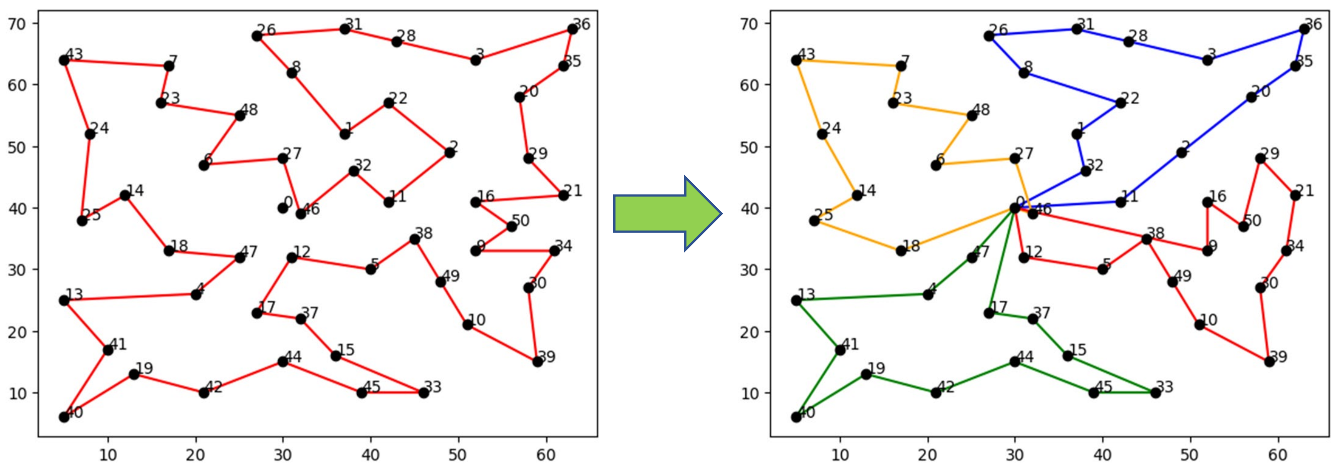


Figure 4. Travel routes obtained using the basic method (total delivery distance; 526.23).



(a) First stage (b) Second stage (total delivery distance; 514.87)

Figure 5. Travel route obtained using the proposed method with the 2-opt method.

5. Conclusion

As a solution to the VRP, this study has proposed methods that find travel routes to all delivery locations using an SOM at the first stage, and among the travel routes found, determine an efficient travel route for each vehicle using the OP and 2-opt methods at the second stage. Then, it assessed the performance of the proposed methods. Findings indicate that the proposed methods facilitate a solution that reduces the total delivery distance more than the basic method. The effectiveness of the proposed methods was verified. Moreover, this study revealed that vehicles' delivery distances tended to have a considerable degree of dispersion. Topics for future studies include improvement of solution accuracy by introducing constraints such as a delivery distance for each vehicle, time limits, and other optimal solutions.

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