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Criterion Referenced Factor Analysis (CRFA): Method and Illustration

Alvah C. Bittner, PhD, CPE

Bittner & Associates
Kent, WA 98042-3532 USA

Corresponding author's Email: drbittner@comcast.net

Author Note: Dr Bittner is a HF/E Researcher-Practitioner long concerned with Methods DT&E as part of Product and System Developments (>50yrs.). He is an ISOES Past-President and -Annual Conference Co-Chair

Abstract: CRFA is a valid method for evaluating relationships between criteria and "factors" initially identified from both the predictor and the criterion variables. Among others, it may be applied to classical problems involving: 1) Changes in complex task ability requirements as a function of practice, 2) Impacts of environmental stressors on personality or performance factors overtime, or 3) *Residential Lifestyle Factor Impacts on Energy-Use* (as herein). CRFA differs from traditional battery developments in its: (a) *Initial inclusion* of both criterion and predictor variables for factor identifications, but (b) Exclusion of criterion in the ultimate calculation of factor-scores. This avoids the vexing confounding of criterion variance in factor score estimates, and ultimately provides for unconfounded analyses of criterion and factor relationships. A "Big-Data" illustration of CRFA is presented that highlights the stability of model results for independent samples across years. The primary model of interest built upon a USA-representative survey (N = 2,165) sample of 17 variables adapted from RECS-2005 (USEIA, 2019). These included16 lifestyle-related and an annual energy use criterion (i.e., LNKWH, *Ln-transformed annual KiloWatt Hours*). Unweighted least squares (ULS) factor analysis revealed a 5- Lifestyle factor solution that accounted for 45.5% of the total variation in the 17-variable set and 45.3% of the 16 less LNKWH. "Lifestyle" factor predictions – subsequently derived by CRFA less LNKWH– are found to be remarkably stable when compared to a similar sample taken 4-years earlier (RECS-2001). Specifically, (1) the proportions of LNKWH variance explained with lifestyle factor scores alone are nearly identical across the 4-year gap (2005 R^2 = 0.42- and 2001 R^2 = 0.38; ps $<10^{-10}$), (2) these increased after external additions of household characteristics (R^2 = 0.55 both fore- and back-casting; ps $<10^{-15}$), and model B-weights were near identical. CRFA is strongly recommended for valid evaluations of relationships between criteria and predictor-based factor-scores, where factor characterizations are initially derived from both predictor and criterion variables.

Keywords: Factor-Analysis Method, Factor-Score Predictor-Criterion Unconfounding, Robustness Illustration

1. Introduction

CRFA *includes both criterion and reference-predictor variables in the initial characterizations of factors but excludes* criterion variables in subsequent estimations of their factor-scores; thus, providing for unconfounded analysis of the relationships between criterion and reference factor (as well as, later added indexing or predictor scores). Illustration of the CRFA process is presented below after an overview of the historical issues it is designed to escape.

1.1 Background

Adams (2014/1989) offered critiques of FAs that included criterion-predictor mixes, particularly mixes of ability and target-task training performances (e.g., Woodrow, 1938; Fleishman & Hempel, 1954). Following Thurstone (1935), Bechtoldt (1962) and others, Adams questioned the appropriateness of "looking at" criterion loadings on a factor that is partially defined by the criterion itself. Pertinently, this critique may be obviated by the simple expedient of blanking/ignoring criterion loadings during factor characterizations. Adam's more salient critique was that there isn't a definition of resulting factors (e.g., factor-scores) not dependent on the criterion. This latter critique led Humphrey (1960), and generations (including ourselves), to recommend separately factor-analyzing criterion and reference-predictor measures (e.g., McCallum, Bittner et al., 2005; Bittner, Croffut & Stranahan, 2005). Very recently, Anderson et al. (2021), noting Adam's critiques and unrelated issues, have unnecessarily suggested moving to a mix of other approaches in future research. CRFA, as will be seen, alternately maintains the frame of the classical approach, but 1) Criterion and index-predictor variables are: 1) ultimately not mixed into the final factor-variable computations, and 2) Criterion and index analyses are subsequently conducted only with factor-scale scores computed strictly from the independent variables.

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1.2 Purpose

This report is designed to provide HF/E practitioners a functional understanding of CRFA: how it may be applied, and its advantages. It draws on an illustrative application (Sanquist, Orr, Shui & Bittner (2012). In addition to the historical frame, the current report also includes extended analytic details developed earlier (Bittner, 2010), as well as incorporates enhancements developed recently (2019-2021). Toward these aims, the basic CRFA processes are overviewed, illustrated, and discussed. The overall goal is to provide a basic framework for future HF/E applications of CRFA.

2. Method & Results

2.1 Overview of CRFA Process

CRFA is a three-step process that differs from a line of recommended developmental FAs (e.g., McCallum et al, 2005, after Humphreys, 1960) in its initial inclusion of criterion and index variables. As with all FAs, data preparations may include selective transformations to minimize collinearities and/or rescaling of multi-categorical scales (e.g., correlated indexing variables).

- ***Step 1 – Conduct principal components analysis (PCA) – including criterion and index variables -- to ascertain the number of dimensions to explore in subsequent factor extraction.*** Traditional extraction approaches often consider both (a) number of PCs with >1 eigen-values and (b) number above "scree level" in a Scree Plot (Harmon, 1975, Norusis, 2011). We have – to date – typically found these traditional approaches adequate (but, see Morrison, 1990, re: alternatives).

- ***Step 2 – Conduct initial factor analyses based upon the identified dimensional numbers.*** We typically employ an Unweighted Least Squares FA (ULS) with a Varamax Rotation (cf., Norusis, 2011). Jöreskog (2003) has advocated this technique – aka, MINRES, PFA with iterated communalities, etc. – as particularly "robust" in that it does not require specific assumptions about the distribution of the data. The results of this initial analysis are informative in several ways. First, one may usually "characterize/identify" the resulting factors from considerations of prominent factor loadings, not including criterion or index variables (i.e., their "blanking"). Second, communalities, including the criterion, are well-known to be conservative estimates of their reliabilities (cf., Harmon, 1975). Of significance, the initial non-criterion variable loadings will be generally better defined and robust than from analyses sans criterion or index variables. Finally, *preliminary* factor-variables may be calculated for each case – e.g., by Least-Squares (Scheffé', 1999/1959; Noruss, 2011) or Ridge-Regression (Hoerl & Kennard, 1988) – together with associated estimates of their reliabilities and covariances (SPSS, 1997; Norusis, 2011). Ridge-Regression is an atypical method for estimating initial factor scores, but it is particularly applicable when data are ill-conditioned, modestly sample sized and potentially subject to: (a) inflated predictor coefficients, and/or (b) coefficient sign changes (Wang & Liu, 2009). This latter issue has 1) long plagued analyses structurally parallel to that addressed herein (e.g., neurobehavioral time courses with chemical exposures in occupational settings), and 2) earlier led to our recommendations of other semi-analytic alternatives (Heyer, Bittner & Echeverria, 1996).

- ***Step 3 – Compute final "criterion/index-free" factor-variables, their associated reliabilities, and co-variances.*** This may be readily accomplished by multiple regressions predicting the initial factor-score variables using only non-criterion variables (this analytically also may be accomplished directly from loadings by ignoring criterion variables when computing factor-scale variables). As with the preliminary factor variables, the reliabilities and covariances may again be assessed from the predicted factor-variable scores. Pertinently, the final-factor prediction-equations are available for applications to other data sets. The final factor-scale variables "sans criterion/index variables" most-interestingly may now be unambiguously related to the criterion measure(s) of interest by multiple correlations as seen in the following illustration.

CRFA is technically completed with these three steps and exploration of the resulting relationships with the criteria. We recommend, however, validations of the results (e.g., fore-and back-casting model results as later illustrated herein).

2.2 Lifestyle Factors Illustration

Our CRFA illustration, as noted above, primarily draws from a report of *lifestyle factors in residential electricity consumption* using 17 variables adapted from RECS-2005 (Sanquist et al, 2012). To later illustrate the robustness of CRFA, we shall later compare results for RECS-2005 and a similar independent sample taken four years earlier, RECS-2001. RECS-2005 provided a USA-wide sample ((N=2,165) of 17 variables: 16 lifestyle-related and an annual energy use metric (i.e., LNKWH). Following subsections successively address: Initial Factor Analysis RECS-2005 (2.2.1), Computation of Unconfounded Factor Scores (2.2.2), and CRFA Results for RECS-2005 with RECS-2001Cross-Comparisons (2.2.3)

Table 1. Five Factor Results for 17 RECS-2005 Variables

VARIABLES	FACTORS					Commu-nality
	AC Factor	Laundry Factor	PC Use Factor	Climate Factor	TV Use Factor	
Scaled Air Conditioning	0.952	0.073	0.084	0.153	-.026	0.943
Natural Log of Total Cooled Square Feet	0.844	0.086	0.055	0.092	0.034	0.732
Clothes Dryer Use, Electric	0.081	0.827	0.050	0.044	0.066	0.699
Clothes Washer Use	0.026	0.731	0.130	-.016	0.259	0.620
Number PCs	0.058	0.080	0.868	-.009	0.042	0.765
PC Time On Total, Weekly	0.057	0.070	0.565	0.017	0.044	0.330
Cooling Degree Days	0.262	-.007	-.073	0.936	0.030	0.951
Heating Degree Days	-.051	0.065	-.022	-.721	-.069	0.532
Number TVs	0.153	0.119	0.295	0.009	0.528	0.404
Number household members	-.042	0.347	0.218	0.016	0.429	0.354
TV Use	-.009	0.104	-.048	0.077	0.355	0.145
Number Ceiling Fans	0.267	0.092	0.143	0.258	0.135	0.185
Dish Washer Use	0.177	0.315	0.329	-.073	0.138	0.263
Hours per day light bulbs are on	0.105	0.137	0.242	0.021	0.173	0.119
Oven Use, Electric	0.101	0.249	0.068	-.030	0.087	0.085
Age of House	-.227	-.123	-.179	-.130	-.043	0.117
LNKWH	0.393	0.348	0.321	0.203	0.262	0.489

2.2.1 Initial Factor Analysis RECS-2005

Using SPSS/*PASW*-18 (cf., Norusis, 2011), a ***Step-1 PCA*** – as outlined above (2.2) – was found to support a 5-factor solution, based on the scree-plot and the number of >1 eigen-value components. Following this, ***Step 2 – a 5-factor unweighted least squares (ULS), with VARIMAX rotation –*** was conducted on the 17 variable correlations. Ridge-Regression not being indicated, initial factor-scores were computed by regression. The initial ULS 5-factor solution altogether accounted for 45.5% of the total variation among 17-variables and 45.3% of the 16 less LNKWH. Table 1 presents ULS results with factors labeled based on their shaded prominent variable loadings (i.e., correlations with respective factors).

2.2.2 Computation of Unconfounded Factor Scores

Step 3, Regression analyses – *using all but LNKWH* – were used to predict the *final factor-scale scores for RECS-2005* (again with SPSS/PASW-18). These clearly resulted in 5 lifestyle factor-scale score estimation equations that are based only on the first 16 lifestyle variables (i.e., without LNKWH). These equations subsequently were used to compute the set of *final* lifestyle factor-scale scores for each of the users analyzed in RECS-2005. Of particular note, the variances of these (N=2,165) final factor-scale scores indicated reliabilities of 0.940 for the AC, 0.771 for Laundry, 0.791 for PC Use, 0.939 for Climate, and 0.470 for TV Use. In a parallel activity (not ordinarily part of Step 3), CRFA process was conducted for an independent sample taken 4 years earlier (i.e., RECS-2001, N= 2,690) and again revealed a 5-factor solution. These final RECS-2005 and -2001 factor-scale-scores provided the basis for comparative evaluations of their concurrent correlations with respective LNKWH.

2.2.3 CRFA RECS-2005 Results And RECS-2001 Cross-Comparisons

Comparative summaries of CRFA independent applications to RECS-2005 and -2001 are provided in Figure 1 and Table 2. Figure 1 shows that the proportions of LNKWH variance explained with lifestyle alone were nearly identical following CRFAs on data sets collected across a 4-year gap ($R^2 = 0.42$ and 0.38; *Ps* $<10^{-10}$), and these became identical after additions of respective household income, natural gas access and price variables ($R^2 = 0.55$ both cases; *Ps* $<10^{-15}$). This parallel – in independent samples across 4-years – will be reflected in their model B-weights as seen in the following.

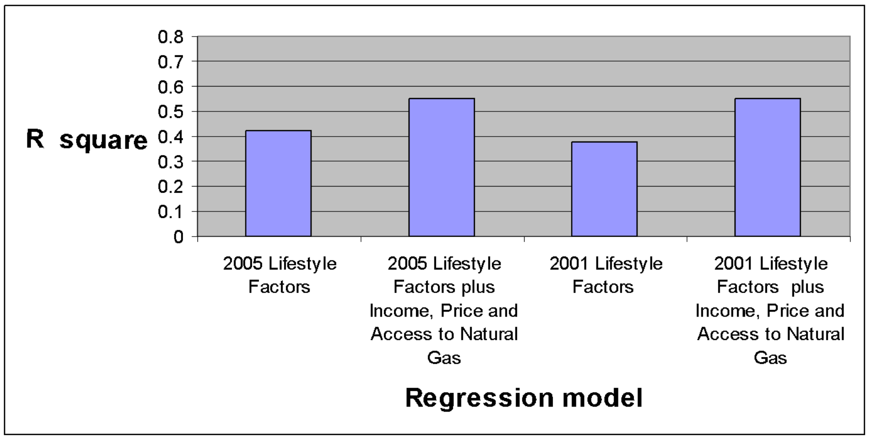

Figure 1. R^2 for Models Predicting Electricity Usage for Lifestyle-Factors and Household Variables.

Table 2 shows the near parallel model coefficients – for the 2005 and 2001 samples and serves to illustrate the potential robustness of CRFA for other applications. Remarkably, despite the substantial power anticipated with "Big-Data" sample sizes (>2K both samples), only the 2001 B-weight for Natural-Gas Ln-Price (-.793) proved just significantly different from the corresponding value (-.699) in 2005 (Z= 2.103; P<0.04, 2-tail). Of note, this relatively meager difference might have been anticipated, given the exceptional increase in KWH-price experienced during the 2001 timeframe (re: 9/11 Attack, Dot.Com Bust etc.).

3. Discussion

CRFA *progressively includes and then excludes* criterion and indexing variables with an aim to enhance developmental factor-analyses (FA). Due to these exclusions from the final factor-scale score computations, it effectively escapes traditional criticisms associated with criterion-predictor intermixing (e.g., Adams, 1989; Bechtoldt, 1962). CRFA, due to its transitory inclusions of criterion and index variables, also possesses advantages over methods with separate criterion-and predictor FAs.

3.1 Advantages

CRFA – including a criterion measure – has three interrelated advantages over a parallel analysis without such an inclusion.

- ***Comprehensiveness.*** Initial criterion inclusion supports identification of single predictor variables that are associated with the criterion, but weakly associated with other predictor variables. Criterion variables also may likewise add support to broader, but still modestly defined factors. Criterion inclusion consequently supports the identification of factors that might be otherwise lost without inclusion.

- ***Factor Number Identification.*** Loadings of an included criterion could be evaluated as successive PCs are extracted during a PCA – both visually and statistically (e.g., Morrison, 1990 Sec 8.6). This can be added to other assessment tools, e.g., Scree Plots and >1 eigen-values (Harmon, 1975).

- ***Variable Reliabilities***. Communalities represent lower-bound estimates of associated reliabilities that only tend to be increased with increases in numbers of 1) extracted factors, and 2) included variables (cf, Harmon, 1975). Consequently, communality reliability representations will be enhanced both by the direct addition of the criterion variable and any additions to numbers of extracted factors (i.e., with increased "comprehensiveness. Finally, the criterion's own communality-reliability estimate may often be estimated only via inclusion in a CRFA.

CRFA advantages were in-part demonstrated in the life-style factor illustration presented earlier. These all together resulted in 1) the identification of *lifestyle factors from the 2005 RECS that were kindred to those in RECS--2001 as well as 2) the development of remarkably stable predictive models.*

3.2 Conclusion

CRFA offers substantial advantages over traditional developmental factor-analyses as has been outlined herein and altogether may be recommended for future instrument developments with targeted criterion and index concerns.

1. Table 2. Unstandardized B-Coefficients for 2005 and 2001 Models

Model Variables	**B-Coefficients**	
	2005	**2001**
Constant	6.937	7.031
Lifestyle Factors		
AC	0.186	0.184
Laundry	0.144	0.156
PC Use	0.164	0.166
Climate	0.136	0.109
TV Use	0.188	0.204
Household Characteristic		
LNKWH Cost	-.699	-.793
Natural Gas	-.291	-.302
Ln Income	0.085	0.054

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